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Investigating The Effect Of Salmonella Discharge From Carrier Individuals In An Epidemic Model Of Typhoid Fever

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Abstract

The simulation of the bacteria discharge impact by carriers in the classes was conducted using a hybrid block method and it was discovered that the bacteria discharge rate by carrier individual and time order of the discharge rate are important factors to be considered in control and eradication of typhoid fever.

Keywords: Hybrid Block Method, Saturated Incidence Rate, Epidemic Model, Disease-Induced Death.

1. Introduction

Mathematical models have been a strategy for controlling and eradicating of several diseases over the years (Momoh, A. A., Ibrahim, M. O. Uwanta, I. J., Manga, S. B. (2013). A comparison of saturation term and induced death in SIR was introduced by Kolawole, M. K., Ogundeji, O. D., Popoola, A. O. (2020). General knowledge, vaccination and treatment was introduced in the work of Kolawole, M. K., Ayoola, T. A., Abdulrazaq, R. (2020). The existence of solution of differential equations of fractional order was presented by Olayiwola, M. O., Gbolagade, A. W., Ayeni, R. O., Mustapha, A. R. (2008).

2. Mathematical Model

The system of equations that described the model is given below:

$$\frac{dS}{dt} = N + \delta R(t) - (\mu + \lambda)S(t)$$

$$\frac{dC}{dt} = \rho\lambda S(t) - (\sigma_1 + \theta + \mu + \phi)C(t)$$

$$\frac{dI}{dt} = (1 - \rho)\lambda S(t) + \theta C(t) - (\sigma_2 + \beta + \mu + \alpha)I(t)$$

$$\frac{dR}{dt} = \beta I(t) + \phi C(t) - (\mu + \delta)R(t)$$

$$\frac{dB_c}{dt} = \sigma_1 C(t) + \sigma_2 I(t) - \mu_b B_c$$

where $\lambda = \frac{B_c}{k + B_c}$.

3. Model Solution and Simulation by the Hybrid Method

The method for solving the problem in differential equation

$$\frac{du(t)}{dt} = f(t,u),\ u(x_0) = u_0 \qquad (1)$$

and its related system

$$\frac{dU(t)}{dt} = f(t,U),\ U(x_0) = U_0 \qquad (2)$$

where $f(t,u)$ is continuous and j^{th} differentiable, was derived by assuming a continuous approximant for $u_n(t)$ of a multi-step multi-derivative method of the form:

$$u_n(t) = \alpha_n(t)y_n + \sum_{i=1}^{l} h^i\left(\beta_{i,0}(t)f_n^{(i-1)} + \beta_{i,v}(t)f_{n+v}^{(i-1)} + \sum_{j=1}^{k} \beta_{j,k}(t)f_{n+j}^{(i-1)}\right) \approx y(t) \qquad (3)$$

where k is the step number, l is derivative order, $v \in (0,k)$ is an off-step point and j is the derivative order of $f(t,y)$. However, the continuous coefficients $\beta_{i,j}(t)$, $i = 0, v, k, j = 1(1)k$ were determined.

Ogunniran (2019) obtained approximation of the exact solution $y(t)$ by evaluating the function:

$$u(t) = \sum_{j=0}^{r+ls-1} a_j t^j \tag{4}$$

where a_j, $(j = 0,1,\dots,r+ls-1)$ are coefficients determined, t^j are the basis functions of degree $r+ls-1$, l, r and s being the derivative order, interpolating and collocations points respectively.

While ensuring that the function (3) corresponds with the analytical solution at the end point t_n, the following conditions were imposed on $u(t)$ and its derivatives $u^{(k)}(t)$ to get the coefficients of the desired methods:

$$\left.\begin{aligned} &u(t_{n+j}) = y_{n+j},\ j = 0 \\ &u'(t_{n+j}) = f_{n+j},\ j \in [0,\cdots,k] \\ &u''(t_{n+j}) = f'_{n+j} = g_{n+j},\ j \in [0,\cdots,k] \\ &u'''(t_{n+j}) = f''_{n+j} = h_{n+j},\ j \in [0,\cdots,k] \\ &\vdots \\ &u^{(k)}(t_{n+j}) = f^{(k-1)}_{n+j},\ j \in [0,\cdots,k] \end{aligned}\right\} \tag{5}$$

The coefficients obtained were substituted into (4) to obtain the continuous coefficients in (4) which were then evaluated at t_{n+k} to obtain the desired methods.

Two-step, Second Derivative and One off-step Method

Here $k = 2$, $l = 2$, and $v = \frac{1}{7}$ is the chosen off-step point.

The new approximation now becomes:

$$u(x) = \alpha_n(x)y_n + h(\beta_{1,0}(x)f_n + \beta_{1,\frac{1}{7}}(x)f_{n+\frac{1}{7}} + \beta_{1,1}(x)f_{n+1} + \beta_{1,2}(x)f_{n+2})$$
$$+h^2(\beta_{2,0}(x)f_n + \beta_{2,\frac{1}{7}}(x)g_{n+\frac{1}{7}} + \beta_{2,1}(x)g_{n+1} + \beta_{2,2}(x)g_{n+2})$$

(3.6)

where $g(x_{n+j}) = f'(x_{n+j}) = u''(x_{n+j})$, $j = 0,\cdots,k$.

Setting $r = 1$ and $s = 4$, the interpolating function now becomes:

$$u(x) = \sum_{j=0}^{8} a_j x^j. \tag{7}$$

Obtaining the first and second derivatives, we have:

$$\begin{aligned} &u'(x) = \textstyle\sum_{j=1}^{8} j a_j x^{j-1} \\ &u''(x) = \textstyle\sum_{j=2}^{8} j(j-1) a_j x^{j-2}. \end{aligned} \tag{8}$$

Imposing the following conditions as obtained from (6):

$$u(x_{n+j}) = y_{n+j},\ j = 0 \tag{9}$$

$$u'(x_{n+j}) = f_{n+j},\ j = 0, \tfrac{1}{7}, 1, 2. \tag{10}$$

$$u''(x_{n+j}) = g_{n+j},\ j = 0, \tfrac{1}{7}, 1, 2. \tag{11}$$

Thus, the required method is given in block method as:

$$\left.\begin{aligned} y_{n+2} &= -\frac{31h^2 g_{n+2}}{845} + \frac{16h^2 g_{n+1}}{45} + \frac{19208h^2 g_{n+\frac{1}{7}}}{7605} + \frac{7}{5}h^2 g_n \\ &+ \frac{781h f_{n+2}}{2197} + \frac{4}{9}h f_{n+1} - \frac{470596h f_{n+\frac{1}{7}}}{19773} + 25h f_n + y_n \\ y_{n+\frac{1}{7}} &= -\frac{15997h^2 g_{n+2}}{66805808160} - \frac{33203h^2 g_{n+1}}{3557705760} - \frac{516419h^2 g_{n+\frac{1}{7}}}{250417440} + \frac{565619h^2 g_n}{395300640} \\ &+ \frac{191117h f_{n+2}}{173695101216} + \frac{9703h f_{n+1}}{304946208} + \frac{3016667h f_{n+\frac{1}{7}}}{39862368} + \frac{5308663h f_n}{79060128} + y_n \\ y_{n+1} &= -\frac{43h^2 g_{n+2}}{81120} - \frac{197h^2 g_{n+1}}{4320} + \frac{333739h^2 g_{n+\frac{1}{7}}}{730080} + \frac{101h^2 g_n}{480} + \frac{539h f_{n+2}}{210912} + \\ &\frac{919h f_{n+1}}{2592} - \frac{19176787h f_{n+\frac{1}{7}}}{5694624} + \frac{385h f_n}{96} + y_n \end{aligned}\right\} \tag{12}$$

The application of (3.12) on the SEIR model, resulted into a system of non-linear algebraic equations whose solutions were obtained using the Newton Krylov's formula.

4. Numerical Results and Simulating Graphs

In the implementation of the hybrid method, a system of nonlinear equations must be solved in order to obtain the desired approximation. To solve these nonlinear systems, a Newton-Krylov solver, nsoli.m or a modified Newton solver, nsold was used. It is worthy to note that the numerical simulation was programmed via MATLAB 9.2 version on a personal computer with the following specifications: HP Pavilion x360 Convertible, Processor-Intel(R) Core(TM) i3-7100U CPU @ 2.40GHz, Installed memory (RAM)- 8.00GB, System Type- 64-bits Operating System, x64-based processor, Operating system- Windows 10. Moreso, computational experiments were done with software optimization and only the points of emphases are shown. The table below shows the definitions and values of parameters used with their sources.

Table 1: Table of Parameters

Parameter	Definition	Value
$S_0(t)$	Susceptible strat value	15
$E_0(t)$	Exposed strat value	10
$I_0(t)$	Infected strat value	13
$R_0(t)$	Recovery strat value	11
d	Disease induced death	
ϵ		0.25
μ	Mortality or death rate	0.3
Λ	Birth rate	49
γ	Rate of immunity loss	0.1
β	Disease transmission coefficient	varied

m_1	Saturation term for susceptible individual	0.1
m_2	Saturation term for infected individual	0.2

The numerical results obtained are presented in graphs and they are s described below:

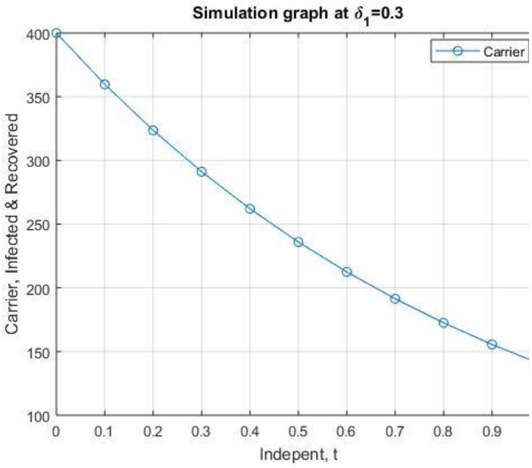

Figure 1: SEIR Simulation for Carrier Graph at $\sigma_1 = 0.3$

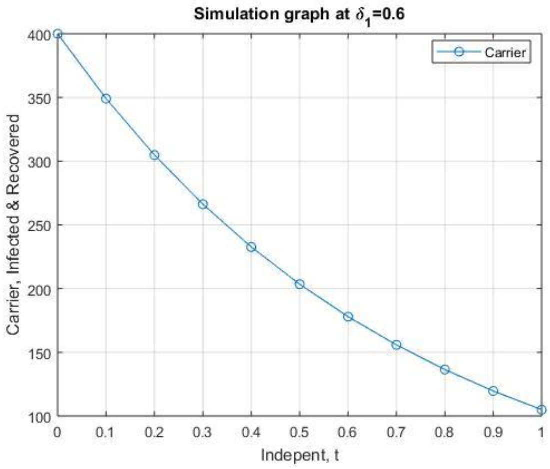

Figure 2: SEIR Simulation for Carrier Graph at $\sigma_1 = 0.6$

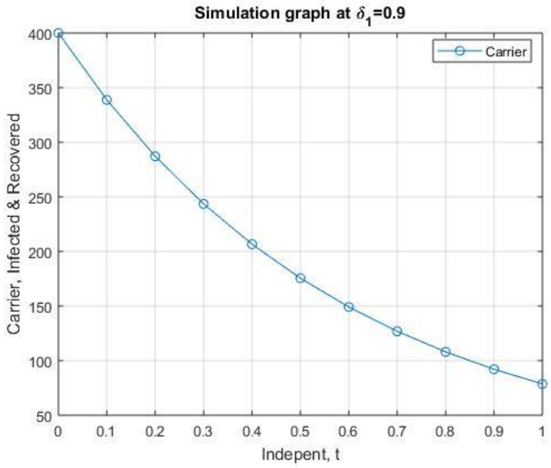

Figure 3: SEIR Simulation for Carrier Graph at $\sigma_1 = 0.9$

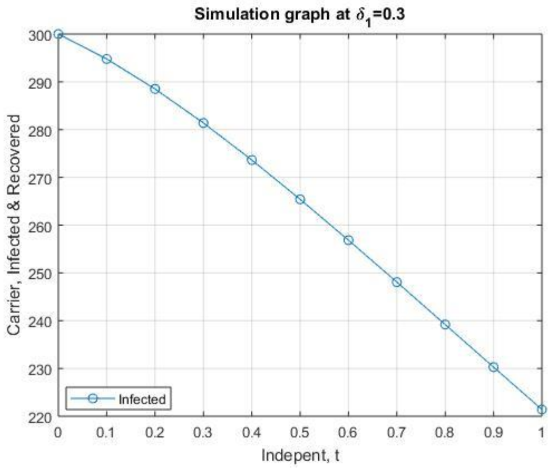

Figure 4: SEIR Simulation for Infected Graph at $\sigma_1 = 0.3$

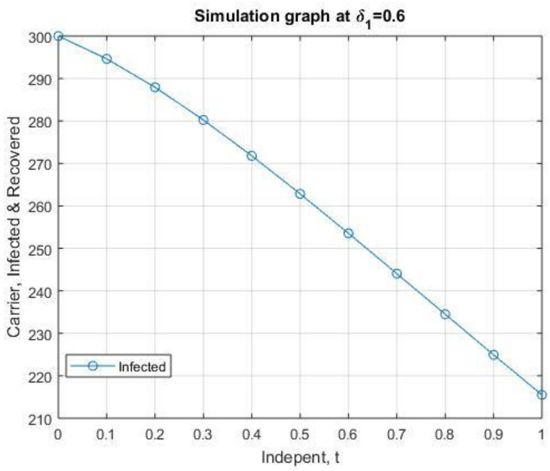

Figure 5: SEIR Simulation for Infected Graph at $\sigma_1 = 0.6$

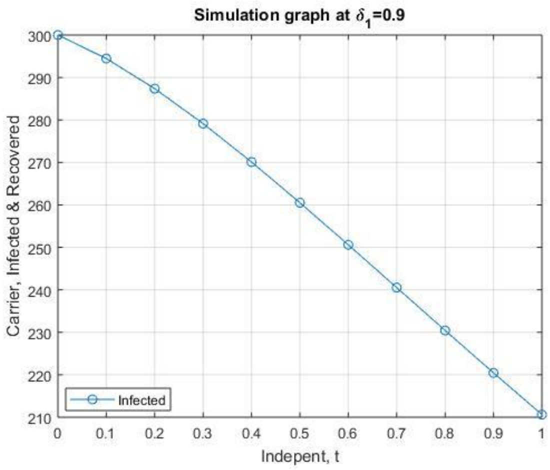

Figure 6: SEIR Simulation for Infected Graph at $\sigma_1 = 0.9$

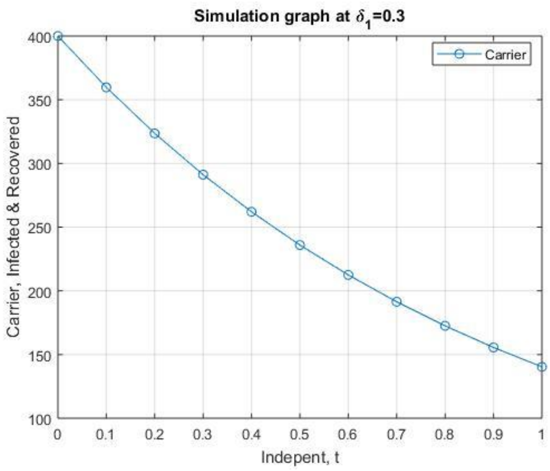

Figure 7: SEIR Simulation for Recovered Graph at $\sigma_1 = 0.3$

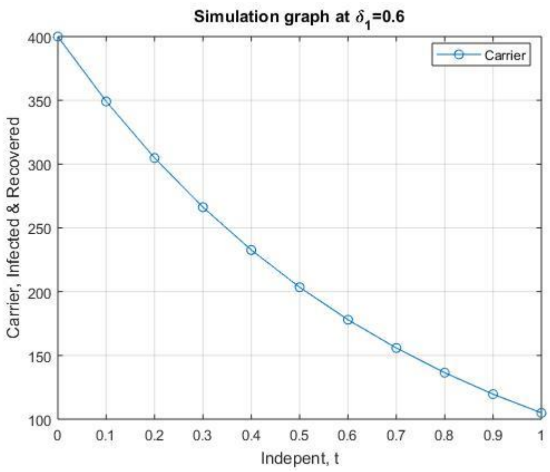

Figure 8: SEIR Simulation for Recovered Graph at $\sigma_1 = 0.6$

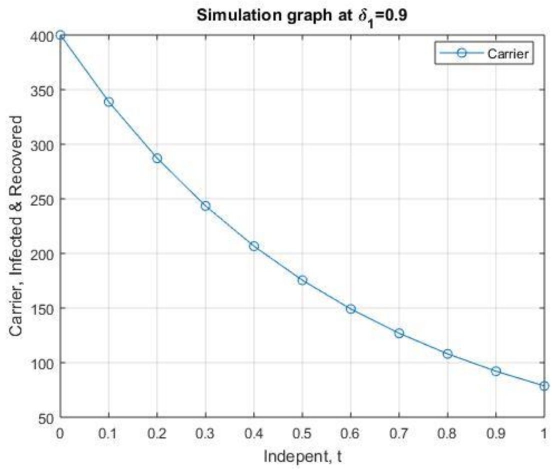

Figure 9: SEIR Simulation for Recovered Graph at $\sigma_1 = 0.9$

5. Discussion of Results and Conclusion

Figures 1-3 show that as the carrier increases from 0.3 to 0.9, the population of infected classes reduced over time. It is also worthy to say that the rate at which the population decreases also depend on time and this is acertain from figures 4-6. It was also noticed from figures 6-9 which represent the recovered class that the population increases due to the introduction of some parmeters of vaccination and immunity. It is also said that people tend to be infected at a higher rate of salmonella discharge σ_1 at a high order.

It is therefore concluded that the salmonella bacteria from infected have a great effect in the spread of typhoid fever. It should be notd that the order of this discharge cannot be under estimated as the rate at which the bacteria discharge occurs plays a significant effect on eventual population of each classes. Hence, these factor is an important measure to consider the eradication of typhoid fever.

Conflicts of interest/Competing interests

The authors declare that there is no conflict of interest null competing interest. All the authors contributed in the preparation of this paper.

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