Journal of Applied Sciences, Information and Computing
Vol. 3, No. 1 (June2022)

ISSN: 1813-3509 https://jasic.kiu.ac.ug

Investigating The Behaviourial Analysis Of Disease Transmission Coefficient In SEIRS Epidemic Model With Disease Induced Death

Kolawole, M. K., Bashiru, K. A. , Ogunniran, M. O., Popoola, A. O.
Department of Mathematicl Sciences, Osun State University, Osogbo Nigeria

Abstract

An extensive behaviourial study and analysis of disease transmission coefficient in a SEIRS epidemic model with disease induced death was carried out in this study. The simulation of the system of linear differential equations was obtained using an effective hybrid block method. The results obtained conforms with the objectives of study.

Keywords: hybrid block method, simulation, epidemic disease, saturation coefficient.

1. Introduction

Numerical solutions to mathematical models have been widely used to understand the spread and control of epidemic disease. The first formulated model was on smallpox and was introduced by Bernoulli (1760). In this paper, one of the hybrid block methods proposed by Ogunniran (2019) is used for the simulation of the SEIRS model to have an insight in the effect of saturation coefficient of the model. The model classifies individual as susceptible, exposed, infected and recovered. Kolawole & Olayiwola (2016), Kunniya and Nakata (2012), Atangana and France (2014) and Merdan et al. (2011) have studied numerous epidemiological models, while in Kolawole & Olayiwola (2016), Kunniya and Nakata (2012), Atangana and France (2014) and Merdan et al. (2011), Marwan et al. (2011) and Rafei et al. (200) numerical methods, including the VIM has been used for the simulation of some models. Recently in Olayiwola (2016a,b), the VIM was used to solve nonlinear differential equations that play vital roles in modeling and simulation.

3. Model Solution and Simulation by the Hybrid Method

The method for solving the problem in differential equation

$$\frac{du(t)}{dt} = f(t,u), \;\; u(x_0) = u_0 \qquad (3.1)$$

and its related system

$$\frac{dU(t)}{dt} = f(t,U), \;\; U(x_0) = U_0 \qquad (3.2)$$

2. Mathematical Model

The system of equations that described the model modification of Kolawole and Olayiwola (2016) is given below:

$$\frac{dS}{dt} = N - \frac{\beta SI}{1 + m_1 S + m_2 I} - \mu S + \delta R$$

$$\frac{dE}{dt} = \frac{\beta SI}{1 + m_1 S + m_2 I} - (\mu + \epsilon)E$$

$$\frac{dI}{dt} = \epsilon E - (\mu + \gamma + d)I$$

$$\frac{dR}{dt} = \gamma I - (\mu + \delta)R$$

where $f(t,u)$ is continuous and j^{th} differentiable, was derived by assuming a continuous approximant for $u_n(t)$ of a multi-step multi-derivative method of the form:

$$u_n(t) = \alpha_n(t)y_n + \sum_{i=1}^{l} h^i\left(\beta_{i,0}(t)f_n^{(i-1)} + \beta_{i,v}(t)f_{n+v}^{(i-1)} + \sum_{j=1}^{k} \beta_{j,k}(t)f_{n+j}^{(i-1)}\right) \approx y(t) \qquad (3.3)$$

where k is the step number, l is derivative order, $v \in (0,k)$ is an off-step point and j is the derivative order of

$f(t,y)$. However, the continuous coefficients $\beta_{i,j}(t)$, $i = 0, v, k$, $j = 1(1)k$ were determined.

Ogunniran (2019) obtained approximation of the exact solution $y(t)$ by evaluating the function:

$$u(t) = \sum_{j=0}^{r+ls-1} a_j t^j \qquad (3.4)$$

where a_j, $(j = 0,1,\ldots,r+ls-1)$ are coefficients determined, t^j are the basis functions of degree $r+ls-1$, l, r and s being the derivative order, interpolating and collocations points respectively.

While ensuring that the function (3.3) corresponds with the analytical solution at the end point t_n, the following conditions were imposed on $u(t)$ and its derivatives $u^{(k)}(t)$ to get the coefficients of the desired methods:

$$\left.\begin{aligned} &u(t_{n+j}) = y_{n+j},\ j = 0 \\ &u'(t_{n+j}) = f_{n+j},\ j \in [0,\cdots,k] \\ &u''(t_{n+j}) = f'_{n+j} = g_{n+j},\ j \in [0,\cdots,k] \\ &u'''(t_{n+j}) = f''_{n+j} = h_{n+j},\ j \in [0,\cdots,k] \\ &\vdots \\ &u^{(k)}(t_{n+j}) = f^{(k-1)}_{n+j},\ j \in [0,\cdots,k] \end{aligned}\right\} \qquad (3.5)$$

The coefficients obtained were substituted into (3.4) to obtain the continuous coefficients in (3.4) which were then evaluated at t_{n+k} to obtain the desired methods.

Two-step, Second Derivative and One off-step Method

Here $k = 2$, $l = 2$, and $v = \frac{1}{7}$ is the chosen off-step point.

The new approximation now becomes:

$$u(x) = \alpha_n(x)y_n + h(\beta_{1,0}(x)f_n + \beta_{1,\frac{1}{7}}(x)f_{n+\frac{1}{7}} + \beta_{1,1}(x)f_{n+1} + \beta_{1,2}(x)f_{n+2})$$
$$+h^2(\beta_{2,0}(x)f_n + \beta_{2,\frac{1}{7}}(x)g_{n+\frac{1}{7}} + \beta_{2,1}(x)g_{n+1} + \beta_{2,2}(x)g_{n+2})$$
(3.6)

where $g(x_{n+j}) = f'(x_{n+j}) = u''(x_{n+j}),\ j = 0,\cdots,k$.

Setting $r = 1$ and $s = 4$, the interpolating function now becomes:

$$u(x) = \sum_{j=0}^{8} a_j x^j. \qquad (3.7)$$

Obtaining the first and second derivatives, we have:

$$\begin{aligned} u'(x) &= \sum_{j=1}^{8} j a_j x^{j-1} \\ u''(x) &= \sum_{j=2}^{8} j(j-1) a_j x^{j-2}. \end{aligned} \qquad (3.8)$$

Imposing the following conditions as obtained from (6):

$$u(x_{n+j}) = y_{n+j},\ j = 0 \qquad (3.9)$$
$$u'(x_{n+j}) = f_{n+j},\ j = 0, \tfrac{1}{7}, 1, 2. \qquad (3.10)$$
$$u''(x_{n+j}) = g_{n+j},\ j = 0, \tfrac{1}{7}, 1, 2. \qquad (3.11)$$

Thus, the required method is given in block method as:

$$\left.\begin{aligned} y_{n+2} &= -\frac{31h^2\, g_{n+2}}{845} + \frac{16h^2\, g_{n+1}}{45} + \frac{19208h^2\, g_{n+\frac{1}{7}}}{7605} + \frac{7}{5}h^2\, g_n \\ &+ \frac{781h\, f_{n+2}}{2197} + \frac{4}{9}h\, f_{n+1} - \frac{470596h\, f_{n+\frac{1}{7}}}{19773} + 25h\, f_n + y_n \\ y_{n+\frac{1}{7}} &= -\frac{15997h^2\, g_{n+2}}{66805808160} - \frac{33203h^2\, g_{n+1}}{3557705760} - \frac{516419h^2\, g_{n+\frac{1}{7}}}{250417440} + \frac{565619h^2\, g_n}{395300640} + \\ &+ \frac{191117h\, f_{n+2}}{173695101216} + \frac{9703h\, f_{n+1}}{304946208} + \frac{3016667h\, f_{n+\frac{1}{7}}}{39862368} + \frac{5308663h\, f_n}{79060128} + y_n \\ y_{n+1} &= -\frac{43h^2\, g_{n+2}}{81120} - \frac{197h^2\, g_{n+1}}{4320} + \frac{333739h^2\, g_{n+\frac{1}{7}}}{730080} + \frac{101h^2\, g_n}{480} + \frac{539h\, f_{n+2}}{210912} + \\ &\frac{919h\, f_{n+1}}{2592} - \frac{19176787h\, f_{n+\frac{1}{7}}}{5694624} + \frac{385h\, f_n}{96} + y_n \end{aligned}\right\} \qquad (3.12)$$

The application of (3.12) on the SEIR model (2.1), resulted into a system of non-linear algebraic equations whose solutions were obtained using the Newton Krylov's formula.

4. Numerical Results and Simulating Graphs

In the implementation of the hybrid method, a system of nonlinear equations must be solved in order to obtain the desired approximation. To solve these nonlinear systems, a Newton-Krylov solver, nsoli.m or a modified Newton solver, nsold was used. It is worthy to note that the numerical simulation was programmed via MATLAB 9.2 version on a personal computer with the following specifications: HP Pavilion x360 Convertible, Processor-Intel(R) Core(TM) i3-7100U CPU @ 2.40GHz, Installed memory (RAM)- 8.00GB, System Type- 64-bits Operating System, x64-based processor, Operating system- Windows 10. Moreso, computational experiments were done with software optimization and only the points of emphases are shown. The table below shows the definitions and values of parameters used with their sources.

Table 1: Table of Parameters

Parameter	Definition	Value
$S_0(t)$	Susceptible start value	15
$E_0(t)$	Exposed start value	10
$I_0(t)$	Infected start value	13
$R_0(t)$	Recovery start value	11
d	Disease induced death	
ϵ		0.25
μ	Mortality or death rate	0.3
Λ	Birth rate	49
γ	Rate of immunity loss	0.1
β	Disease transmission coefficient	varied
m_1	Saturation term for susceptible individual	0.1
m_2	Saturation term for infected individual	0.2

The numerical results obtained are presented in graphs and they are s described below:

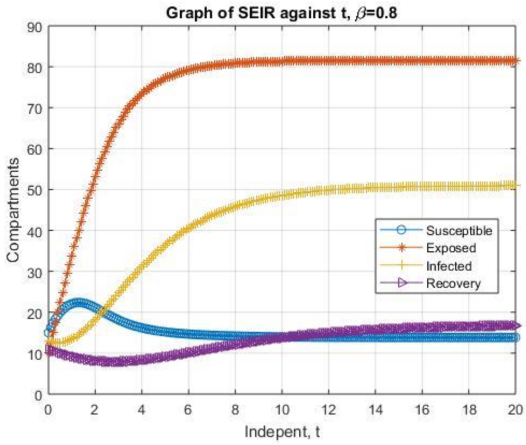

Figure 1: SEIR Simulation Graph at $\beta = 1.0$

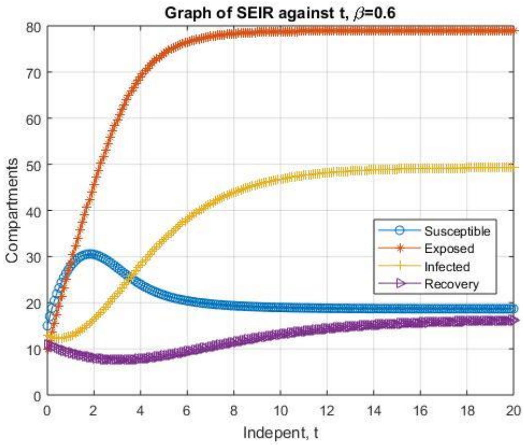

Figure 2: SEIR Simulation Graph at $\beta = 0.75$

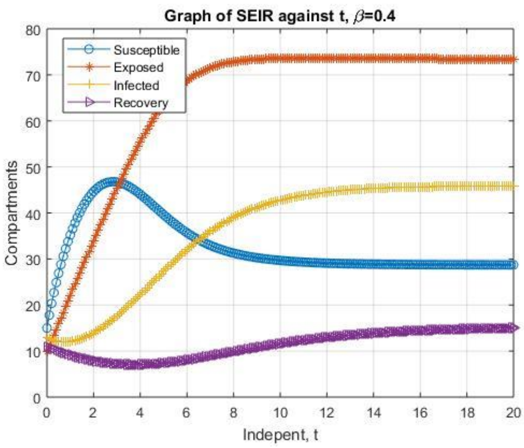

Figure 3: SEIR Simulation Graph at $\beta = 0.5$

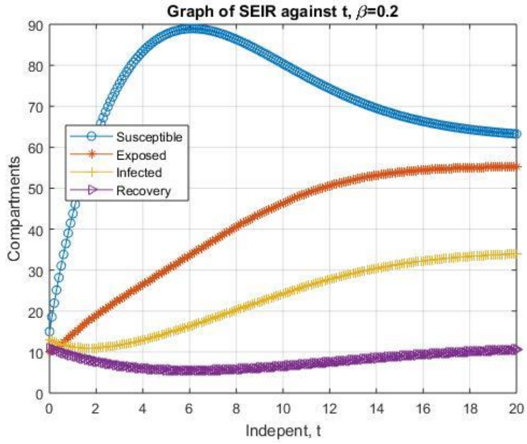

Figure 4: SEIR Simulation Graph at $\beta = 0.35$

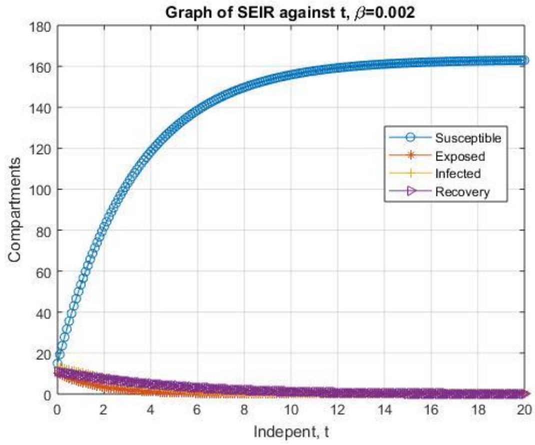

Figure 5: SEIR Simulation Graph at $\beta = 0.001$

5. Discussion of Results and Conclusion

Figure 1 displays the simulating result when β=1.0, the exposed class increases with infected class, while the susceptible and recovered class were at least. Figure 2 shows the simulation at β=0.75 and it was observed that the exposed class is also on the increase. However, figures 3-5 reveals drastic changes in susceptible class. It is therefore concluded that the lower the value of β, the better the stability of the disease-free equilibrium. It is therefore concluded that in the presence of a permanent immunity, β plays a vital role in disease eradication.

Conflicts of interest/Competing interests

The authors declare that there is no conflict-of-interest null competing interest. All the authors contributed in the preparation of this paper.

Acknowledgments

The authors would like to express their profound gratitude to anonymous referees for their contributions and helpful advice.

References

Adebimpe, O., Wahed, A. A. and Gbadamosi, B.: Modeling and analysis of an SEIRS epidemic model with saturated incidence. Int. Journal of Engineering Research and Application, 1111-1116, (2013).

Atangana A., Emile France, D. E. (2014). Biomed Research International, 214(1).

Bernoulli, D. (1760). Essai d'une nouvelle analyse de la mortalite causee par la petite verole et sed avantages de l'inoculation pour la prevenir. In Memoires de Mathematiques et de Physique. Academie rovale des sciences.

Kolawole, M. K. & Olayiwola, M. O.: Simulation of the Effect of Disease Transmission Coefficient on the Susceptible-Exposed-Infected-Recovered (SEIR) Epidemic Model Using Variational Iteration Method. Anale Seria Informatica, XVII, 339-343 (2019).

Kolawole, M. K. and Olayiwola, M. O.: Analysis of a SEIRS Epidemic model with saturated incidence rate considering the initial state of the disease. Transactions of NAMP 2, 171-180 (2016).

Kolawole M. K. and Olayiwola M. O. (2016). Computing Information Systems, Development Information and Allied Research Journal, 7(2), 83.

Kunniya, T and Nakata Y. (2012). Applied Mathematics and Computing, 218(18), 9321.

Merdan, M, et al. (2011). Computers and Mathematics with Applications, 62(118).

Ogunniran, M. O.: A Class of Block Multi-derivative Numerical Techniques for Singular Advection Equations. J. Nig. Soc. Phys. Sci. 1, 62-71 (2019).

Olayiwola, M. O., Kolawole, M. K. and Popoola, A. O.: Variational Iteration Method for the simulation of Disease Transmission Coefficient on the SEIRS epidemic model with saturated incidence rate and disease induced death. Journal of Art and Science 2(39): 357-364 (2017).

Olayiwola M. O. (2016). Journal of Science and Art (Mathematics) (36), 243.

Rafei, M. et al. (2007). Applied Mathematics and Computation, 186(2), 1707.